The Relation between Tracking Error and Tactical Asset Allocation

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Abstract

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In this paper we investigate the relation between statistical tracking error measures and asset allocation restrictions expressed as admissible weight ranges. Typically, tracking errors are calculated as annual standard deviations of return differentials between tracking portfolio and benchmark. In practice, however, constraints on tactical deviations from benchmark weights are often imposed instead on the portfolio manager to ensure adequate tracking. Simulating various investment strategies subject to such constraints, we illustrate how the size of acceptable deviations from the benchmark relates to the statistical tracking error. Using an example based on actual market data, we find that imposing fairly large tactical asset allocation ranges implies surprisingly small tracking errors. Another finding of the paper is that tactical asset allocation restrictions should not only restrict the tactical ranges of the individual asset classes, but even more importantly, the tracking of the individual asset classes.

1. Introduction

This paper addresses an important problem in practical asset management: the relationship between statistical tracking of a portfolio and tactical asset allocation ranges.

Typically, tracking errors are calculated as annual standard deviations of return differentials between tracking portfolio and benchmark. In practice, however, constraints on tactical deviations from benchmark weights (subsequently called "tactical ranges") are imposed on a portfolio manager to ensure adequate tracking and limit the active portfolio risk. These bounds define the maximum percentage deviations by which the actual portfolio weights may deviate from the corresponding weights in the benchmark portfolio. For example, an equally weighted benchmark portfolio including five asset classes exhibits strategic weights of 20% for each class; an active management contract may allow the portfolio manager to deviate from these strategic weights within a range of plus/ minus 10% for each class. This implies a tracking error, which gives the active manager the chance to earn abnormal portfolio returns, i.e., achieve a positive alpha.

A natural question is how statistical tracking error measures, as defined in Section 2, relate to the structure and size of these range constraints. We analyze tracking errors allowed by different choices of the underlying tactical ranges for various assumptions regarding the underlying portfolio strategy.

The analysis of the relationship between statistical tracking error and allocation constraints defined in terms of weight ranges is of high practical relevance because analysts, investment strategists, and risk managers often think in terms of tracking volatility or correlation, whereas the actual allocation decisions by portfolio managers tend to be guided by recommendations and constraints on the weights of assets or asset classes in their portfolios.

Throughout the paper, we take a simulation approach to analyze these questions: for given tactical asset allocation ranges, we identify admissible tactical portfolio combinations and simulate return time series for these portfolios, based on historical data from international stock and bond markets. We then use these time series to calculate correlation and tracking error of the simulated portfolios.

The paper is structured as follows. In the next section, the tracking error measures used in this paper are described. Section 3 gives a statistical characterization of the returns used in the subsequent simulations. In section 4, static deviations from a benchmark portfolio are analyzed. In contrast, section 5 allows portfolio managers to implement dynamic strategies in the underlying asset classes. In section 6, it is assumed that the active returns obtained on the individual asset classes are different from the corresponding returns in the passive benchmark portfolio. Section 7 relates the tracking error to performance fees in asset management. Section 8 summarizes the findings of the paper.

2. Tracking Error Measures

Tracking errors can be expressed by a variety of statistical measures. For example, the correlation coefficient is a straightforward tracking measure. Other popular tracking measures include first or second moments of the deviations between portfolio and benchmark returns. As a first tracking error measure, we use the square root of the non-central second moment of these deviations, i.e.,

(1) $$TE1 \equiv \sqrt{\frac{\sum_{k}^{n}(R_{Pk} - R_{Bk})^2}{n-1}}$$

where R_{Pk} denotes the return of the tracking portfolio in period k, R_{Bk} the return of the predetermined benchmark portfolio in period k, and n the sample size. TE1 is a tracking error measure that is frequently used in practice. The tracking portfolio P can be an *active* portfolio where the assets weights change dynamically (tactical asset allocation), or a *passive* portfolio where the asset weights do not change over time but are different from the corresponding weights of the benchmark portfolio.

The previous measure can be interpreted as standard deviation, but because it is a non-central measure, not only random positive and negative deviations affect this measure but also a possible constant out- or underperformance relative to the benchmark. Another popular tracking error measure is the centered volatility as proposed by Roll (1992). In this paper, however, we use the uncentered version to capture not only the volatility of return deviations, but also their extent.

Alternatively, the tracking error of a portfolio can be defined as the residual volatility of the tracking portfolio with respect to the benchmark, as proposed by Treynor and Black (1973). Specifically, the ex post tracking error of a tracking portfolio can be computed as the standard deviation of the residuals of a linear regression between the tracking returns and those of the benchmark portfolio, i.e.,

(2) $$TE2 \equiv \sigma(\varepsilon_P) = \sigma(R_P) \cdot \sqrt{1-\rho_{PB}^2}$$

where $\sigma(R_P)$ is the volatility of the tracking portfolio and ρ_{PB} denotes the correlation coefficient between the returns of the portfolio and the benchmark. For example, if the portfolio volatility is assumed to be 20%, a correlation coefficient of 0.95 implies a tracking error of 6.24%. This figure can be interpreted easily and has direct applications in portfolio risk calculations. It can also be used to compute risk adjusted performance measures (e.g., the Black/ Treynor (1973) appraisal ratio) or to determine the expected return on active strategies (i.e., in the context of Grinold's law of active management, see Grinold/ Kahn 1995).

The classical tracking error problem concentrates on minimizing tracking error when replicating a benchmark portfolio under restrictions, such as replicating with an incomplete set of securities. Examples for a treatment of this tracking error problem include Rudd (1980), Larsen and Resnick (1998), Rudolf, Wolter, Zimmermann (1999), among others. This paper, however, addresses the relationship between tracking errors caused by active portfolio management and given tactical asset allocation ranges.

3. Descriptive statistics of returns and benchmarks

The subsequent analysis uses a sample benchmark portfolio for a US investor including the following asset classes: US bonds, Canadian bonds, Japanese stocks, US stocks and European stocks. The benchmark is an equally weighted portfolio in these asset classes. The bond returns are measured by Salomon Brother bond indices, the stock returns are based on the

Morgan Stanley total return indices, all measured in USD. The descriptive statistics of the asset class returns, as perceived by an USD based investor, are displayed in Table 1. The time period for which the returns are measured is from January 1985 to June 1998. Among the asset classes, the Japanese stock market exhibits the lowest returns and the highest volatility. The benchmark portfolio has an average return of 13.5% and a volatility of roughly 10% over the sample period.

4. Static tactical asset allocation

In this section, the portfolio manager chooses a passive asset allocation within the pre-determined ranges. The ranges determine the maximum permissible deviations from the benchmark weights for the individual asset classes. Because we have specified an equally weighted benchmark, the tactical ranges are also assumed to be equal across the asset classes, for simplicity. We specify ranges of (plus and minus) 5%, 10% and 20%; we also investigate the case where no restrictions are imposed (except that the weights can neither be negative nor exceed 100%). Short positions are excluded throughout the paper. The sum of the risky portfolio holdings must add up to unity, i.e., there is no borrowing or lending.

A search procedure is used to identify all possible portfolio allocations satisfying the constraints imposed by the specified tactical allocation ranges. In this context, we are particularly interested in those strategies that exhibit the lowest correlation (or respectively, the highest tracking error) with respect to the benchmark for a given tactical range. To limit the number of admissible portfolios, the steps by which the portfolio weights can be modified are assumed to be 5%, 10% and 20%, depending on the width of the tactical bands. This generates the following number of tactical portfolio combinations:

Allocation range	*Step size*	*Number of portfolios*
5%	5%	51
10%	5%	381
20%	10%	381
No restriction	20%	126

The resulting portfolios are ranked according to the correlation coefficients with respect to the benchmark. Table 2 gives the five portfolios with the lowest correlation for each allocation range constraint. Columns 3-7 give the portfolio weights in %, column 8 gives the correlation coefficient of the portfolio returns with respect to the benchmark returns, column 9 tracking error measure TE1, column 10 tracking error measure TE2, and column 10 Jensen's alpha of the portfolio with respect to the benchmark portfolio.

If a deviation range of 5% is imposed, the lowest attainable correlation with the benchmark is 0.9886 in our sample. For a 10% range the lowest correlation coefficient is 0.9464, for a 20% range only 0.692. Without constraints it drops to 0.4284. With strategic weights of 20%, a tactical range of (plus/ minus) 10% may be considered as fairly large in the sense that it gives an active portfolio manager substantial flexibility to over- or underweight individual asset classes. Therefore, a correlation coefficient of 0.9464 intuitively appears surprisingly high (or correspondingly, a tracking error of 3.3% or 2.7% appears low). Reversing the argument implies that even "narrow" statistical tracking error constraints give active managers a fair amount of flexibility to implement their strategies.

The ranking by correlation does not correspond to the ranking by TE1 or TE2. Because TE2 does not only depend on correlation, but also on the volatility of the portfolio, the correlation ranking does not correspond to the tracking error ranking. For example, although portfolio 2 (only Canadian bonds) among the unrestricted portfolios in Table 2 exhibits a higher correlation with the benchmark than portfolio 1 (only U.S. bonds), its tracking error is larger because of the higher volatility of Canadian bonds relative to U.S. bonds during the sample period. Similarly, correlation and TE1 rankings do not coincide because the extent of the deviation from the benchmark return changes the TE1 measure but not the correlation coefficient.

It can also be seen that TE1 and TE2 give quite different results. Because TE1 measures the total volatility (uncentered) of the deviation, it is always larger than TE2, which contains only the volatility of the residual return.

Overall, the results show that moderately passive portfolios, i.e. narrow asset allocation ranges, exhibit almost perfect correlation between portfolio and benchmark. Only when ranges are fairly wide do tracking errors become substantial. Note that the figures reported in Table 2 are the *minimum* correlation coefficients consistent with the imposed ranges. The range and distribution of tracking errors and correlation coefficients of *all* permissible portfolios for a given range constraint are graphically displayed in Figure 1.A-D. For each portfolio, the tracking error TE1 (left axis) and the correlation coefficients (right axis) are shown. The portfolios are listed in descending order with respect to TE1.

The Jensen's alphas of the extreme portfolios are negative or positive depending on the performance of the portfolios that have minimal correlation with the benchmark. In our sample, the alphas of the portfolios with low correlation tend to be fairly high and positive. This is, however, not a general result. A different data sample may have resulted in negative alphas for the low-correlation portfolios. The alpha values are included for illustrative purpose only and do not influence our tracking error analysis in any way. It may be interesting to note, however, that the average alpha computed over all admissible portfolio combinations is always slightly negative. The negative values increase for wider ranges. This result is intuitive because, as diversification is lost because of undiversified portfolios, underperformance becomes more likely.

Figure 2 shows distributions of TE2 values for ranges +/-10%, +/-20%, and without restrictions. Small or zero tracking errors are possible regardless of range constraints. The wider the allocation ranges, however, the higher are mean and variance of the tracking error distribution. While a range of +/-10% implies a tracking error of 3% or less, a range of +/-20% implies tracking errors as high as 6%.

5. Dynamic tactical asset allocation strategies

In the previous section we assumed that the tactical allocation remained unchanged over the entire investing horizon. The purpose of this section is to investigate the effect of dynamic allocation on tracking error figures. We therefore assume that the portfolio manager follows a dynamic allocation strategy, changing portfolio weights each month subject to the constraints given by the tactical allocation ranges. We investigate three different types of allocation strategies:

- Random re-balancing. Each month, the portfolio weights are randomly chosen from the total number of tactical allocations satisfying the range constraints.
- Re-balancing based on a trend-following strategy. Again, the tactical weights are randomly chosen subject to the range constraints, but two additional constraints are added: The weight of the asset class with the highest return in the previous month cannot decrease in the following month and must be at least as high as its weight in the benchmark. This implies that the weight of the best performing asset from the previous period always equals or exceeds its benchmark weight in the following period. This strategy is trend-following in the sense that it tends to favor the “winning” asset class from the previous period.
- Tracking error maximization strategy. Each month the portfolio weights are set such that the TE1 measure is maximized while still satisfying the range constraints.

Of course, all three strategies are rather arbitrary; they are not expected to be superior to any alternative strategies. The purpose is to investigate whether a dynamic reallocation between the asset classes has a substantial impact on the tracking results reported in the previous section. The maximizing strategy gives an upper bound on the tracking error attainable for the given sample and reallocation frequency (once a month).

For the examples in this section, the range constraints for the strategy are set to the 10% range. With a step size of 5%, there is a total of 381 available portfolio combinations in each month for the dynamic strategies. The random strategy arbitrarily chooses one of these portfolios in each period. The non-random strategy possibly allows less than 381 combinations because additional constraints are imposed. The maximizing strategy, finally, gives only one portfolio in each period that maximizes the tracking error.

A total of 100 simulation runs is performed for each of the randomized strategies. One simulation run is a complete dynamic allocation series from the beginning to the end of the sample period. The correlation figures for all simulated allocation series are shown in Figure 3. Again, the strategies are ordered in descending order with respect to their correlation vis-à-vis the benchmark.

The correlation values range from 0.9706 to 0.9842 and from 0.9705 to 0.9858 for random and non-random dynamic strategies, respectively. Relative to the static allocations for the 10% range, as displayed in Figures 1.B and 2, the range of correlation values is much narrower in the dynamic case. At times, dynamic allocations differ little from the benchmark allocation, while, at other times, they differ much more from it. This averaging effect and the random nature of the dynamic allocations result in overall correlation values that are somewhere between the extreme values as they occurred for static strategies.

Because it is conceivable that dynamic strategies can be devised that exhibit higher tracking errors than the ones resulting from the two strategies above, we also implement a strategy that maximizes the tracking error (TE1). This maximizing strategy gives an upper bound on the tracking error for our data sample. Table 3 shows the tracking errors obtained with the maximizing strategy.

The highest tracking error (TE1) that can be obtained for our data sample is 2.31% for the 5% tactical range, 4.61% for the 10% range, 9.32% for the 20% range, and 21.12% for the unconstrained case. Table 3 also displays the correlations of the portfolios with maximum tracking error. Note that those correlations can be higher than the minimum correlations identified in the corresponding static allocations as shown in Table 2. Because the ranking of the portfolios with respect to correlation is different from the ranking with respect to tracking error, minimizing tracking error does not result in allocation strategies that maximize correlation.

For comparison, the rightmost column of Table 3 shows the maximum tracking errors (TE1) of the static allocations as discussed in the previous section. Again, the portfolios resulting in these tracking errors do not generally correspond to the portfolios with the lowest correlation shown in Table 2. If the largest tracking errors of the static strategies are compared to the largest attainable tracking errors obtained by the maximizing strategy, one finds that the dynamic procedure does not increase the tracking errors substantially. Dynamic reallocation of assets under given range constraints is therefore not likely to substantially increase tracking error compared to a static allocation policy. It is more likely, as shown by the random allocation strategies displayed in Figure 3, that dynamic allocation averages out extreme tracking errors and results in a “typical” as opposed to extreme tracking error.

6. Static deviations from the benchmark and noisy asset class returns

The results in the two previous sections indicate that selecting "typical" tactical allocation ranges implies strategies with surprisingly high correlation coefficients and low tracking errors with respect to a passive benchmark; this is even true for the most extreme strategies within the given tactical weight constraints. It was assumed so far that the actual portfolio strategy perfectly replicates the performance of each of the underlying asset classes. This is often not true in practice. Portfolio managers mostly follow an active strategy within the individual asset classes, i.e. they deviate from the composition of the individual benchmarks. Therefore, the "passive" returns used to characterize the asset class returns so far must be substituted by returns including an additional "active" component.

As in the previous section, an equally weighted benchmark is used, and tactical ranges of +/-10% are assumed. Assuming portfolio weights as multiples of 5% implies a total of 381 tactical strategies. The returns of these strategies are substituted by active, "noisy" returns r_{At}. They are generated by the following scheme: First, the passive asset class returns r_{Pt} are standardized according to $\tilde{z}_{Pt} = (\tilde{r}_{Pt} - \mu_P)/\sigma_P$ where μ_P and σ_P denote mean and standard deviation of the passive return series. Standardized *active* returns are generated according to

(3)... $$\tilde{z}_{At} = \rho\, \tilde{z}_{Pt} + \sqrt{1-\rho^2}\, \tilde{\varepsilon}_t$$

where ε_t represents a time series of random numbers generated by a standard normal distribution, and ρ denotes a pre-specified correlation coefficient between active and passive returns and thus represents the "tracking" of the respective asset class. Transforming the standardized returns back leads to

(4)... $$\tilde{r}_{At} = \tilde{z}_{At}\sigma_P + \mu_P$$

A total of 381 tactical strategies is generated and ordered according to their correlation coefficient with respect to the passive benchmark. This simulation is performed for 3 sets of correlation coefficients, 0.9, 0.8 and 0.7, respectively. The results are displayed in Figure 4.

Comparing the results to those of Section 3 reveals substantially lower correlation coefficients. In order to facilitate the comparison, the case of “passive” returns is also displayed in the Figures (“no noise”). In this case, the strategy with the highest tracking error exhibits a correlation coefficient of 0.9464. These coefficients sharply decrease if the passive returns are substituted by moderately active returns (represented by a correlation coefficient of 0.9): the lowest correlation coefficients are 0.88 and 0.93, respectively. As is apparent from the Figure, the impact on the correlation coefficients is virtually the same across the strategies. The effects of a further increase of the active component of asset returns (represented by correlations of 0.8 and 0.7) are also displayed in Figure 4. Overall it is apparent that the tracking of the individual asset classes has a more pronounced effect on the correlation between an actively managed portfolio and the benchmark than the selection of the tactical weights. For example, in Figure 4, the entire spectrum of “no noise” strategies satisfying the imposed tactical allocation range of 10% produces correlation coefficients within a narrow range of 0.05 (from 0.95 to 1). If the asset class returns are tracked with a correlation of only 0.7, however, the correlation decreases by 0.15 to 0.3 to values between 0.67 and 0.85, depending on the allocation. Additionally, the range of possible correlation coefficients widens to approximately 0.18 (values from 0.67 to 0.85.)

The practical implication is that restrictions for tactical asset allocation should not only restrict the *weight ranges* of the individual asset classes (i.e. the determination of tactical ranges) as often done in practice, but more importantly, the *tracking of the individual asset classes*.

7. Robustness of the Results

The results presented in the previous sections refer to a specific time period and a specific benchmark portfolio. To investigate the robustness of our findings, we replicate part of the results for two subperiods and for a different benchmark portfolio.

The descriptive statistics of the asset classes for the two subperiods are given in Table 5. Some of these statistics vary substantially over the two subperiods, the most extreme case being the return on Japanese stocks that changes from nearly +20% in the first subperiod to nearly –3% in the second subperiod. Volatility is lower in the second subperiod on all markets with the exception of the Canadian bond market. The correlation of the Japanese stock markets with all other markets including bond markets decreases substantially in the second subperiod while the correlation between U.S. and European stock markets decreases only slightly. The only increasing correlations are between U.S. stock and U.S. bond markets and between U.S. stock and Canadian bond markets.

Given these descriptive statistics, we expect tracking correlations to be slightly lower in the second subperiod. This does not mean, however, that tracking errors TE1 and TE2 need to be higher because the lower market volatility in the second subperiod tends to decrease the tracking error. These conjectures are confirmed in Table 6. Tracking correlation is slightly lower. On the other hand, the effect of a lower volatility in the second subperiod is sufficient to override the effect of slightly lower correlations, and results in smaller tracking errors in the second subperiod. Overall, however, the order of magnitude of tracking correlations and tracking error remains unchanged.

As an additional test of the robustness of our results, we change both the benchmark portfolio and the time period. Japanese stocks and Canadian bonds are replaced with Swiss stocks and Swiss bonds, respectively. Moreover, the time period is extended, now ranging from December 1980 to June 1998. These changes imply quite different descriptive statistics of asset categories and portfolios, as can be seen in Table 7. As a consequence, tracking correlations and tracking errors are also different. Comparing the new tracking errors in Table 8 with Table 2 shows that, for the new benchmark, correlations are somewhat higher, tracking errors lower, but within the same order of magnitude as the values of Table 2. Table 8 also shows that the relation between tracking error and tactical allocation ranges is very similar to that observed in Table 2 for a different benchmark portfolio. These results indicate that the observations and interpretations in the previous sections will likely remain valid for other benchmark specifications and time periods.

8. An Economic Interpretation of the Tracking Error: The Value of Performance Fees

A natural way to give tracking errors an economic interpretation is by determining the implied value of performance fees. Kritzman (1987) shows that performance fees that are based on a benchmark portfolio exhibit the structure of exchange options. An active portfolio manager who participates from the positive excess performance of a portfolio, and earns a fixed fee otherwise, actually owns an exchange option that gives him the right to exchange the benchmark portfolio against the active portfolio he is managing. The number of exchange options he owns depends on his participation schedule. The economic value of the performance fee contract can be used to determine the appropriate discount from the percentage fixed fee that would be used otherwise.

The value of an exchange option W can be determined by the Margrabe (1978) model. In our setting the formula is

$$W = V_P N(z_1) - V_B N(z_2)$$

where

$$z_1 = \frac{\ln\left(\frac{V_P}{V_B}\right) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \quad z_2 = z_1 - \sigma\sqrt{T},$$

$$\sigma^2 = Var(\ln V_P - \ln V_B) = \sigma_P^2 + \sigma_B^2 - 2\rho_{PB}\sigma_P\sigma_B$$

V_P is the current value of the portfolio under management, V_B the value of the benchmark portfolio, and σ_P and σ_B are the volatilities of the respective log portfolio changes. ρ_{PB} is the correlation coefficient between the log changes of the portfolio and the benchmark. T denotes the maturity of the options. N(.) denotes the cumulative standard normal distribution. In our setting, we select $V_P = V_B$, and the maturity equal to one year. The volatilities and the correlation coefficient are those from the portfolio with the *maximum* tracking error within the specific tactical range.

Table 9 displays the value of exchange options related to our tracking errors. If an active portfolio manager would *fully* participate in the excess performance of his portfolio, and assuming 10% tactical portfolio ranges, then the fair value of the performance fee would be 1.32% of the portfolio value (on an annual basis). A typical performance fee contract might fix a participation of 20% of the excess performance, for example. So the appropriate discount to be applied with respect to an otherwise fixed fee is $0.2 \times 1.32\% = 0.33\%$. Specifically, if a flat fee of 1% is the relevant alternative for the sponsor, then the flat fee should be reduced to 0.67% under the 20%-participation contract. The interesting observation from Table 9 is that the value of the exchange option, and hence of the performance fee, is roughly proportional to the tactical portfolio range.

9. Conclusion

The accuracy of tracking a benchmark can be quantified with various tracking error measures, such as the correlation coefficient between tracking portfolio and benchmark, the volatility of return differentials, or the volatility of residual returns. When active portfolio strategies are implemented in practice, however, investors often find it more convenient to specify bounds on tactical deviations from benchmark weights defined for the various asset classes. This paper illustrates the relationship between the size of these tactical ranges and their corresponding statistical tracking error measures. For this purpose, we simulate all possible tactical portfolio holdings satisfying certain pre-specified range constraints. The simulation is based on a portfolio of international stocks and bonds from the perspective of a US investor and the associated historical returns.

For given ranges, we demonstrate that the lowest attainable correlation coefficients between the tactical portfolios and the benchmark are surprisingly high. A further finding is that the correlation coefficients are more sensitive to the tracking accuracy of the individual asset classes. This implies that restrictions imposed to control the deviation of tactical asset allocation strategies from benchmarks should not only restrict the *weighting* of the individual asset classes (i.e. the determination of tactical ranges) as often done in practice, but also the *tracking* of the individual asset classes.

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Table 1: Asset classes and benchmark portfolio characteristics

Asset Class	Mean	Volatility	Correlation Coefficients				
	In %	In %	B-US	B-CAN	MSCI-JAP	MSCI-US	MSCI-EUR
B-US	9.49	5.48	1				
B-CAN	9.85	8.92	0.52	1			
MSCI-JAP	8.14	25.41	0.11	0.13	1		
MSCI-US	17.12	14.83	0.28	0.29	0.21	1	
MSCI-EUR	19.20	16.45	0.24	0.22	0.46	0.62	1
Equally weighted benchmark	*13.51*	*9.85*	*0.42*	*0.46*	*0.76*	*0.69*	*0.82*

Table 2: Static deviations from benchmark

Tactical Ranges	Five most extreme portfolios with respect to correlation						Corr. With Bench.	Tracking Error 1 in %	Tracking Error 2 in %	Alpha In %
	#	B-US	B-CAN	MSCI -JAP	MSCI -US	MSCI -EUR				
5%	1	20	25	15	25	15	0.9886	1.65	1.36	0.60
	2	20	15	15	25	25	0.9888	1.50	1.48	0.86
	3	20	25	25	15	15	0.9889	1.50	1.48	-0.88
	4	25	20	25	15	15	0.9891	1.48	1.45	-0.83
	5	15	20	15	25	25	0.9894	1.48	1.45	0.81
10%	1	20	30	10	30	10	0.9464	3.31	2.72	1.18
	2	30	30	10	20	10	0.9495	3.57	2.38	0.96
	3	25	25	10	30	10	0.9496	3.26	2.60	1.23
	4	25	30	10	25	10	0.9501	3.37	2.49	1.07
	5	30	20	10	30	10	0.9510	3.26	2.53	1.27
20%	1	40	40	0	20	0	0.6920	7.14	4.76	1.78
	2	30	40	0	30	0	0.7299	6.74	4.98	2.02
	3	40	30	0	30	0	0.7359	6.68	4.73	2.12
	4	20	40	0	40	0	0.7450	6.62	5.45	2.25
	5	40	20	0	40	0	0.7488	6.53	5.07	2.44
No cons- traint	1	100	0	0	0	0	0.4284	9.06	4.95	1.55
	2	0	100	0	0	0	0.4660	9.79	7.89	0.56
	3	20	80	0	0	0	0.4890	9.14	6.76	0.80
	4	80	20	0	0	0	0.4901	8.69	4.81	1.40
	5	40	60	0	0	0	0.5082	8.73	5.81	1.02

Average alpha: 5% range: -0.01%, 10% range: -0.02%, 20% range: -0.09%, unrestricted: -0.26%.

Table 3: Tracking Error Maximizing Strategy

Tactical Ranges	Dynamic Strategies				Static Strategies
	Correlation to Benchmark	Max Tracking Error 1* in %	Implied Tracking Error 2 in %		Max TE1 in %
5%	0.9723	2.31	2.27		1.79
10%	0.8900	4.61	4.46		3.57
20%	0.8140	9.32	8.91		7.16
Unconstrained	0.7521	21.12	18.11		18.99

** Maximization of the tracking error is done with respect to Tracking Error 1; the correlation coefficient and Tracking Error 2 refer to this maximizing strategy.*

Table 4: Static deviations from benchmark and noisy returns on asset classes

Tracking of Asset Classes (Correlation)	Five most extreme portfolios						Correlation to Benchmark	Tracking Error In %
	#	B-US	B-CAN	MSCI-JAP	MSCI-US	MSCI-EUR		
90%	1	30	25	25	10	10	0.8758	4.29
	2	20	30	30	10	10	0.8769	4.52
	3	25	30	10	25	10	0.8774	3.46
	4	20	30	10	30	10	0.8810	3.80
	5	30	20	30	10	10	0.8817	4.48
80%	1	30	15	30	15	10	0.7778	5.79
	2	25	25	30	10	10	0.8000	5.82
	3	20	30	25	10	15	0.8026	5.67
	4	30	10	30	10	20	0.8036	6.01
	5	25	15	30	20	10	0.8101	5.94
70%	1	25	30	25	10	10	0.6725	5.70
	2	25	15	20	30	10	0.6848	5.49
	3	20	30	25	15	10	0.6866	5.81
	4	30	15	30	15	10	0.6984	6.35
	5	15	20	20	15	30	0.7064	6.24

Table 5: Asset class characteristics for subperiods

Asset Class	**Mean**	**Volatility**	**Correlation Coefficients**				
01/01/85-09/01/91			**B-US**	**B-CAN**	**MSCI-JAP**	**MSCI-US**	**MSCI-EUR**
B-US	11.02%	6.05%	1.00				
B-CAN	12.86%	8.62%	0.66	1.00			
MSCI-JAP	19.33%	27.53%	0.12	0.19	1.00		
MSCI-US	16.32%	17.66%	0.26	0.28	0.24	1.00	
MSCI-EUR	21.50%	20.12%	0.25	0.26	0.52	0.64	1.00
10/01/91-06/01/98							
B-US	7.99%	4.85%	1.00				
B-CAN	6.88%	9.18%	0.37	1.00			
MSCI-JAP	-2.92%	22.84%	0.07	0.05	1.00		
MSCI-US	17.92%	10.94%	0.34	0.32	0.17	1.00	
MSCI-EUR	16.92%	11.87%	0.23	0.16	0.36	0.58	1.0

Table 6: Static deviations and tracking error for subperiods

10% Tactical Range	Five most extreme portfolios with respect to correlation (lowest correlation)						Corr. With Bench.	Tracking Error 1 in %	Tracking Error 2 in %
	#	B-US	B-CAN	MSCI-JAP	MSCI-US	MSCI-EUR			
Full period	1	20	30	10	30	10	0.9464	3.31	2.72
	2	30	30	10	20	10	0.9495	3.57	2.38
	3	25	25	10	30	10	0.9496	3.26	2.60
	4	25	30	10	25	10	0.9501	3.37	2.49
	5	30	20	10	30	10	0.9510	3.26	2.53
Subperiod 01/01/85-09/01/91	1	20	30	10	30	10	0.9532	3.69	2.94
	2	25	25	10	30	10	0.9545	3.69	2.87
	3	30	20	10	30	10	0.9549	3.71	2.83
	4	30	30	10	20	10	0.9566	4.09	2.53
	5	25	30	10	25	10	0.9573	3.80	2.65
Subperiod 10/01/91-06/01/98	1	20	30	10	30	10	0.9321	2.91	2.50
	2	30	30	10	20	10	0.9347	3.00	2.24
	3	25	30	10	25	10	0.9353	2.90	2.33
	4	25	25	10	30	10	0.9390	2.81	2.32
	5	15	30	10	30	15	0.9416	2.70	2.42

Table 7: Asset classes and benchmark portfolio characteristics for alternative benchmark

Asset Class	Mean	Volatility	Correlation Coefficients				
	In %	In %	B-US	**B-CH**	**MSCI-CH**	MSCI-US	MSCI-EUR
B-US	9.50	13.09	1				
B-CH	5.32	3.33	0.15	1			
MSCI-CH	15.50	16.84	0.38	0.28	1		
MSCI-US	14.32	20.67	0.74	0.12	0.65	1	
MSCI-EUR	14.42	17.93	0.51	0.15	0.77	0.76	1
Equally weighted benchmark	*12.32*	*11.87*	*0.75*	*0.25*	*0.83*	*0.92*	*0.90*

Data: 12/1/1980-6/1/1998

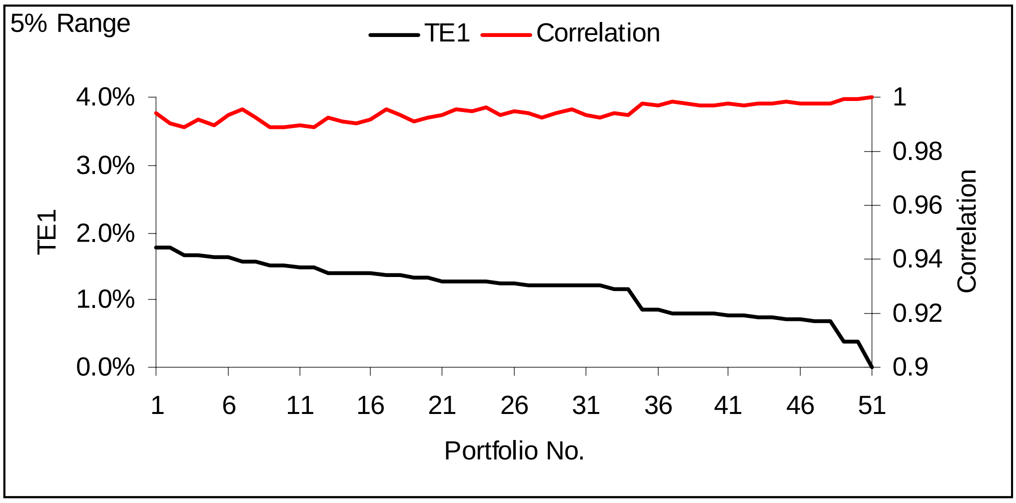
Table 8: Static deviations from alternative benchmark

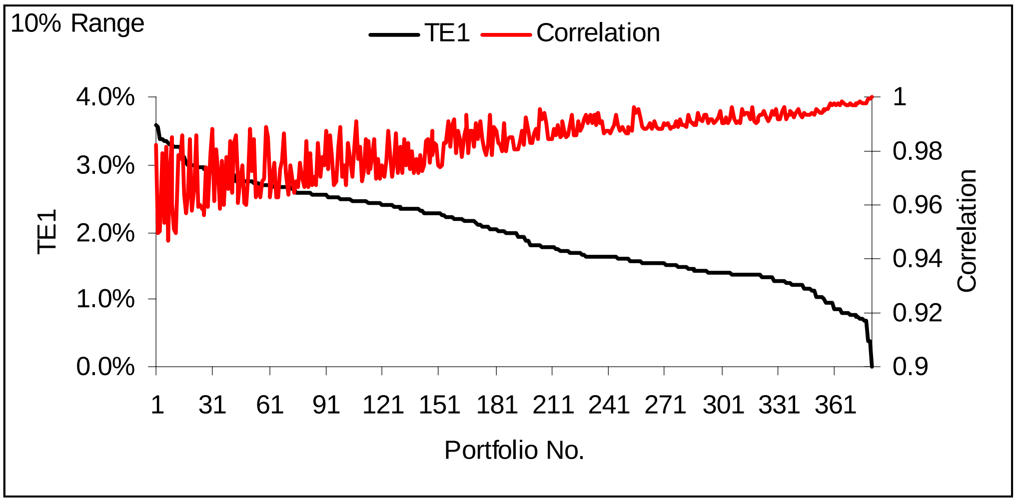
Tactical Bands	Five most extreme portfolios with respect to correlation (lowest correlation)						Correlation to Benchmark	Tracking Error 1 In %	Tracking Error 2 In %
	#	B-US	**B-CH**	**MSCI-CH**	MSCI-US	MSCI-EUR			
5%	1	25	20	1.28	25	15	0.9942	1.28	1.28
	2	15	20	1.28	15	25	0.9943	1.29	1.28
	3	15	25	1.10	15	20	0.9951	1.31	1.10
	4	25	25	1.04	20	15	0.9954	1.45	1.04
	5	15	15	1.10	25	20	0.9962	1.31	1.10
10%	1	30	20	2.56	30	10	0.9771	2.57	2.56
	2	10	20	2.56	10	30	0.9780	2.57	2.56
	3	10	30	2.21	10	20	0.9785	2.62	2.21
	4	25	25	2.29	25	10	0.9784	2.53	2.29
	5	30	30	2.08	20	10	0.9786	2.89	2.08
20%	1	40	40	4.15	20	0	0.8875	5.75	4.15
	2	0	40	4.42	0	20	0.8989	5.24	4.42
	3	40	30	4.57	30	0	0.9064	5.06	4.57
	4	0	30	4.70	0	30	0.9143	4.87	4.70
	5	40	20	5.12	40	0	0.9152	5.14	5.12

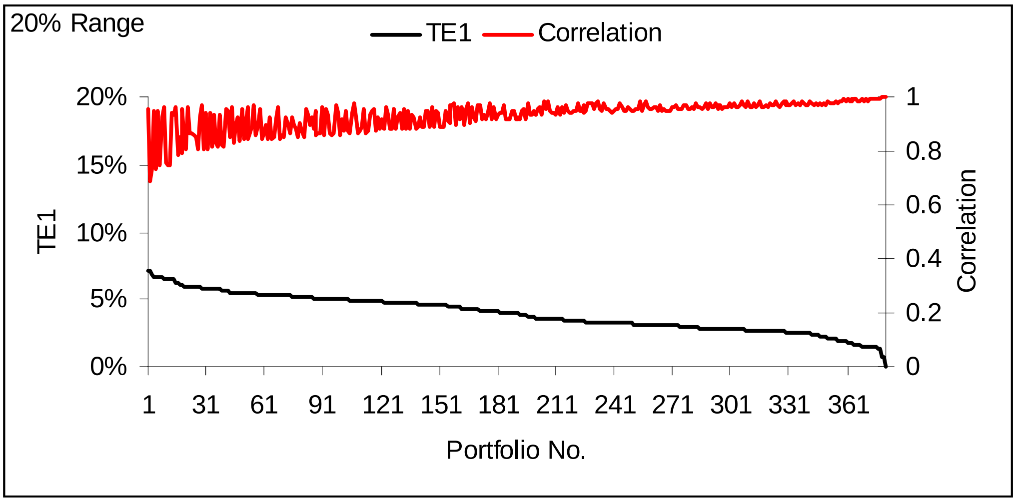
Table 9: Values of the exchange option

Tactical Portfolio Range	Volatility of Benchmark Portfolio	Volatility of Active Portfolio	Correlation Coefficient	Value of Exchange Option (in % of Portfolio Value)
	σ_B	σ_P	ρ_{BP}	W
5%	9.85%	9.01%	0.989	0.65%
10%	9.85%	8.42%	0.946	1.32%
20%	9.85%	6.60%	0.692	2.84%

Figures 1.A-D: TE1 and correlation between benchmark and passive TAA strategies






20% Range
TE1
Correlation
20%
15%
10%
5%
0%
1
0.8
0.6
0.4
0.2
0
1 31 61 91 121 151 181 211 241 271 301 331 361
Portfolio No.

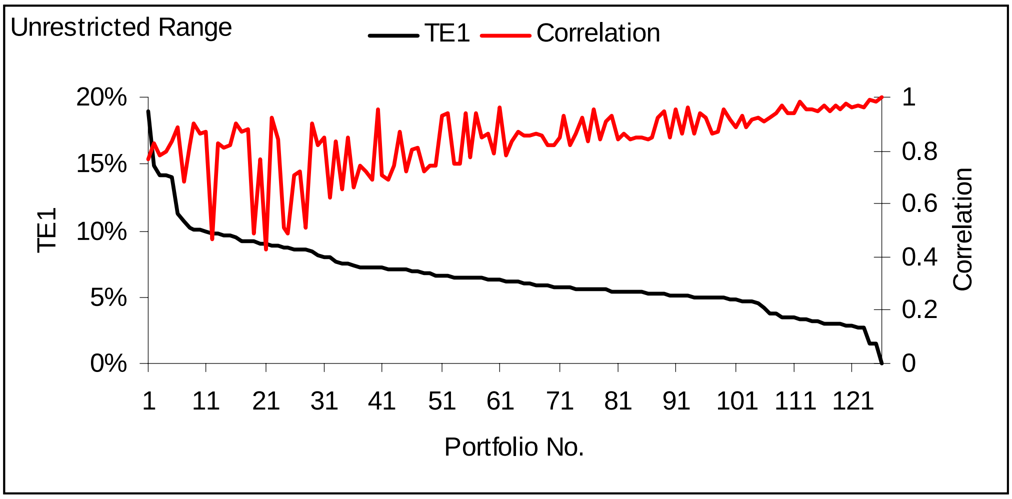

Unrestricted Range
TE1
Correlation
20%
15%
10%
5%
0%
1
0.8
0.6
0.4
0.2
0
1 11 21 31 41 51 61 71 81 91 101 111 121
Portfolio No.

Figure 2: Tracking error (TE2) distributions for passive TAA strategies

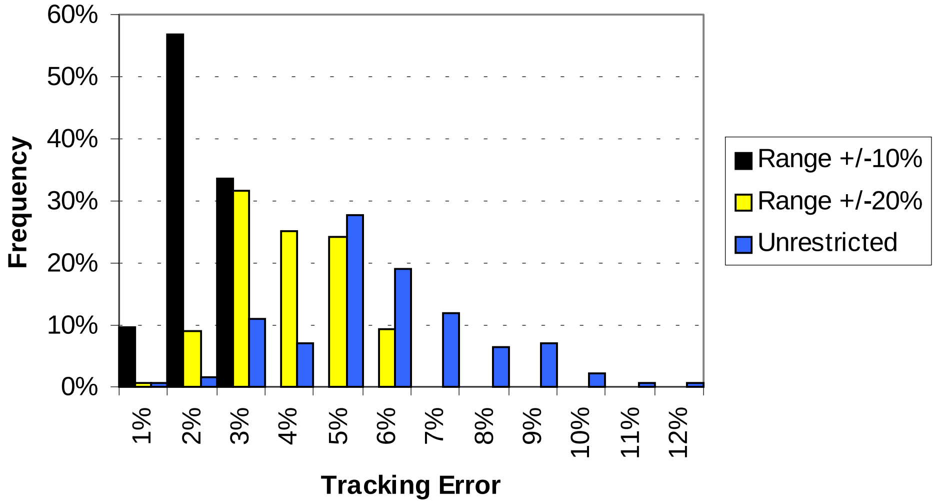


Figure 3: Correlation between benchmark and dynamic TAA strategies

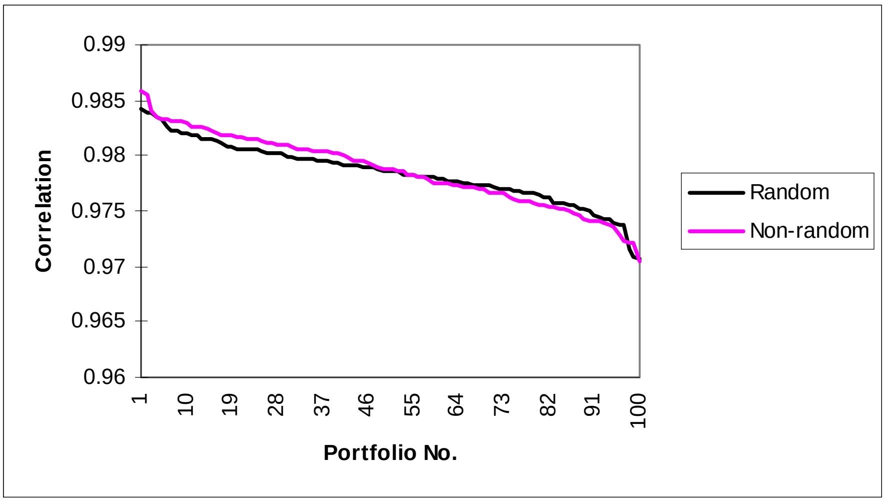


Figure 4: Correlation between benchmark and noisy TAA strategies